**Innovative Method of the Evaluation of Multicriterial Multicast Routing Algorithms**

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**Abstract**—Theoretical considerations of the multicast Quality of Service (QoS) routing have been a rapidly developing and dynamic research area for years. Several algorithms derived from different approaches have been proposed, while the pool of valid solutions to the problem is steadily growing. When new solutions are compared with their predecessors, as much information as possible about their characteristics and differences is needed. Both the graph theory and the optimization theory provide robust and objective means of comparing not only algorithms, but also the results they produce. However, any possible extension to the comparison methods is vital and can bring interesting new information that would eventually lead to innovative conclusions. This article presents a method, derived from practice and experience, that simulates the drainage of resources accumulated by consecutive communication allocations. The nature of this comparison is an extension to the classical measurement of the success ratio and this creates a context of the continuous measure of a success rather than a simple binary value. In this article such a method with regard to algorithms optimizing multicast problems for more than two criteria is used for the first time and leads to an interesting conclusion about the influence of the number of the criteria on the result.

**Keywords**—evaluation, graph algorithms, multicast, QoS, resource drainage, routing.

1. Introduction

The concept of QoS is the foundation of the process of network convergence. A multitude of services can be provided over the network with the use of a single medium because their requirements are often disjoint. For example, data transfer services may easily coexist with the narrowband real time traffic as the former mainly require large bandwidth, whereas the latter are mostly satisfied with just stable delay guarantees.

One of the more popular techniques in modern networks is the multicast transmission. It enables simultaneous communication of a group of users which, when properly implemented, may offer great resource savings as compared to the basic point-to-point communication based approach. The real time multicast transmission of multimedia content is a widely-used traffic type, which is a challenging research subject as there is a great demand for it in the rapidly developing area of multimedia telecommunications.

The model considered in the article is the **Constrained Minimal Steiner Tree Problem (CMSTP)**. [1], [2] that involves connecting a single source with multiple destinations in such way that one of the multiple metrics of the structure is minimal, under the restriction that the others do not violate respective constraints. Therefore, when comparing different algorithms, one has to examine the costs of the multicast tree found in a given graph for given input parameters. The evaluation of the result is a non-trivial task. The metric which is to be minimized should obviously be the lowest, but the constrained metrics may be of greater or lesser importance depending on assumed goals. For example, from the user point of view, any result that satisfies the constraints will be acceptable. It may even be advantageous if the resulting constrained metrics are significantly lower than the proposed constraints. This may, however, lead to an excessive resources drainage which is harmful for the service provider.

From the provider’s point of view, the higher the constrained metrics, the better (provided that the constraints are not violated) as it allows providers to save their valuable resources. In this article, the provider’s point of view is taken, and so the resources savings process is marked as the main goal. In order to achieve this, an unorthodox comparison technique is to be used. Instead of measuring trees metrics, a special resource drainage scenario has been simulated. In the article, the multicriterial algorithms are compared in this way, and the results of different numbers of criteria are then compared to show how the properties of a given algorithm change with the number of the metrics to be considered.

The article starts with an overview of the available algorithm evaluation techniques and places the one presented by the authors in Section 2. Section 3 introduces a mathematical model used for a description of the algorithms’ input and output, which also constitutes the definition of the considered CMSTP problem. In Section 4, the algorithms that have been compared are characterized briefly and the rationale behind the selection of these particular algorithms as the representatives is also provided. Sections 5 and 6 present the experiment description and the presentation and discussion of the obtained results, respectively. Finally, Section 7 concludes the article.
2. Means of Algorithm Comparison

2.1. Evaluation Criteria

The classical purpose of the graph optimization is to find paths, trees or other sub-graphs of the lowest cost. This requirement naturally leads to the cost of the resulting structure as the comparison criterion. In the case of problems with a reasonable complexity, we usually consider algorithms that guarantee finding an optimal solution and, therefore, the running time complexity is the key to evaluate the algorithms’ quality [3], [4]. This kind of comparison is one of the fundamental concepts of the optimization theory.

If an \( NP \) problem is considered, such as the CMSTP [1], then optimal solutions are in general non-reachable by any dependable means and thus the computational complexity, while still important, is no longer the only determinant factor. The desired solutions are imminently suboptimal, but the goal is to reach the ones that are possibly closest to the optimum. In order to deal with the algorithm evaluation within limited knowledge, relative values such as the differences between the quality metrics of the results are used. Such an approach is very popular in practice and is presented in [5], among others. What is more, feasible solutions to such problems may not be always readily available and, therefore, often the success rate [3], [6] or the deviation of the actual value from the constraint [3], [7], [8] are to be additionally considered.

As an extension to the aforementioned typical ways of evaluating graph algorithms, another approach is presented in [9]. It simulates the depletion of graph resources under an infinite load of multicast connection requests. The objective is to set up multicast trees for randomly selected node groups one after another, increasing the cost of the occupied edges after each allocation. If a cost of any edge grows beyond a certain limit reflecting the complete depletion of its throughput, the edge is removed from the graph. This is performed until the graph connectivity is broken, after which point the graph is no longer considered valid. The result of the simulation is the number of the trees that were allowed to be set up by the algorithm before the graph became disconnected. Ref. [9] presents the methodology for the optimizations of two criteria only.

One of the advantages this approach gives is the relevance to the real life situations in which dynamical structures are considered and the resources management is important throughout a long period of time. The approach also allows improvement to the success rate measurement. In the case in which two algorithms lead to feasible solutions, the classical approach will judge them equally efficient. However, in our approach further allocations are requested so that we can measure and compare continuous measures of the success instead of a binary value.

As an innovation, in this article multiple criteria are considered and the dependency of the results on the number of the considered criteria are presented in the relevant section.

2.2. Problem Properties

There is a number of important parameters of the experiments that describe the problems solved by the evaluated algorithms.

A very important factor is the size of test topologies. Running times directly depend on this parameter, but it may also impact the algorithm procedures indirectly, which is only visible when results for an increasing number of network nodes are presented.

In addition, statistical and topological properties of graphs should be taken into account as there exist a lot of means of obtaining random topologies [10]–[14] and each of them is better suited to reflect different real life networks [15], [16].

This article considers the constrained problems, therefore there is one more important aspect to the graph problems, which is picking constraints so that they are well suited for the comparison. If the constraints are too strict, not many results will be found, if any, and therefore their statistical quality is going to be low unless great amount of computational effort is put into obtaining a sensibly large sample of valid results. On the other hand, if the constraints are too loose, many of the algorithms obtain feasible results early, without any need to perform stronger optimizations, which makes it harder to expose their unique properties. Article [8] presents a technique for picking a single constraint based on a scalar indicating the “toughness” of the problem within the range of \((0, 1)\), 0 or less meaning unsolvable problem, and 1 or more meaning a problem that may be solved without any particular optimization with regard to the constrained metrics. In this article, the method has been generalized to include multiple criteria, and this multidimensional variant has been used to generate the problems in the simulations for this article.

Another factor determining how hard the problems are is the size of the multicast group to be connected. It not only affects the complexity of the computations, as most of the algorithms’ running times depend directly on the number of multicast participants, but also impacts the amount of the resources that is drained from the graph after each tree has been set up.

3. Mathematical Description of the Problem

We model communication network as an undirected graph \( G(N,E) \) defined as a finite set of nodes \( N \) and a set of edges \( E \subseteq \{ (u,v) : u,v \in N \} \), each of which reflects a physical point-to-point link. With each of the edges, we associate a set of \( M \) metrics modeled with real valued functions: \( m_i : E \to \mathbb{R}, i = 0, 1, \ldots, M - 1 \). For each of the metrics except the first we define the constraints \( C_i, i = 1, 2, \ldots, M - 1 \).

We define a path as a sequence of non-repeated nodes \( n_1, n_2, \ldots, n_k \in N \) such that for each \( 1 \leq i < k \) an edge
In this article we evaluate algorithms of the multi-constrained path optimization problem (MCOP), which can be reduced to finding a path $p^*$ such that:

$$\forall_{p \in P(s,t)} m_0(p^*) \leq m_0(p),$$  

where $P(s,t)$ is a set of the feasible solutions, i.e., all the paths in the graph $G$ between the nodes $s$ and $t$ that fulfill the following condition:

$$\forall_{i \in \{1,2,...,M-1\}} m_i(p) \leq C_i.$$  

4. Evaluated Algorithms

4.1. HMCMC

The Heuristic Multi-Constrained MultiCast (HMCMC) algorithm [3] represents a purely multicriterial multicast algorithm. It is based on a two-pass modified Dijkstra’s algorithm in which both the passes utilize a non-linear cost definition. The first of the passes is performed from the destination to all the other nodes in the graph. In this way, a set of labels is defined for each node describing its heuristically defined distance to the destination node. If the tree that is formed this way satisfies all constraints in the paths towards all of the receivers then it is accepted as a final result. Otherwise for each of the destinations that have not been connected to the source via a feasible path, another pass is performed aimed at the optimization of the connection between the particular pair of nodes. The computations for the specific paths are done with use of the information gathered in the initial pass so the results are of better quality than the initial ones at the cost of an additional path finding algorithm run.

4.2. Aggregated MLARAC

In order to demonstrate the discriminating qualities of the presented comparison technique, the algorithm of a very different nature has been selected as the contrasting example. A multicriterial unicast Multi-dimensional LAgrangian Relaxation based Aggregated Cost algorithm (MLARAC) [17] has been chosen as its base. In this class of algorithms, the source node is connected with all destinations one by one, resulting in a collection of paths. These paths are then merged into a single subgraph that is, in turn, pruned in order to remove potential cycles from the structure. Such an approach has been earlier demonstrated in [18]–[20], however only two criteria were involved, whereas the MLARAC algorithm handles an arbitrary number of criteria and is used in such an aggregated form for the first time in this article.

4.3. Aggregated HMCOP

In order to provide better exploration of the aggregated unicast algorithms another unicast algorithm is introduced. Heuristic MultiConstrained Optimal Path HMCOP [6] is a non-linear Lagrangian relaxation based multicriterial path optimization algorithm. The authors introduce a new, non-linear cost function, which is then used in a two pass Dijkstra’s algorithm based search. The first step plays the role of the precomputation providing information for the second pass so that it may efficiently chose good, heuristic result.

5. Experiment Description

The comparison of the multicriterial algorithms is a hard task not only because of the complexity of the algorithms themselves, but also because of the multitude of detail involved in the performance of the simulation, let alone its initiation. All the parameters that were considered in the experiments were broken into two main categories: the fixed and the variable arguments. The fixed arguments are the assumptions we have chosen experimentally in order to most efficiently expose the searched quantities. The variable arguments are the ones that build up the set of the resulting charts, i.e., the multidimensional results’ space.

5.1. Fixed Parameters

Several minor decisions had to be made in order to perform the experiments. Drainage arguments. The parameters for the drainage simulation were based on the solutions from the OSPF protocol [21] that provided the translation between the edge’s cost and the parameters of the underlying physical link:

$$\text{throughput}_{ij} = \frac{C_{\text{min}} C_{\text{max}}}{C_{ij}},$$

where $C_{\text{min}}$ and $C_{\text{max}}$ are the borders of uniform distribution range, and $C_{ij}$ is the cost of the link between node $i$ and $j$. OSPF uses $10^8$ in the numerator, though, based on the actual topologies used in the simulation, we experimentally chose $10^4$. For each stream of data flowing through a link we assumed the drainage of 10 Mbit/s of throughput.

Degree of toughness. A special procedure was used to determine the constraints for the simulated problems. It is presented in [8] and, then, generalized for the multidimensional problems in this article. The coefficient of 0.9 was chosen, which in the scale from 0.0 to 1.0 reflects relatively easy problems. The value was defined arbitrarily in order not to limit the result counts too much so that the differences between the algorithms could be better seen.

Number of graphs. To guarantee the statistical quality, 300 graphs were picked randomly to be considered in each of the major simulation case, which guaranteed the confi-
dence intervals two orders of magnitude less than the obtained average values.

5.2. Experiment Variables

Four of the considered simulation parameters were selected as the variables for the presentation of the results. These are:

- the topology generation algorithm,
- number of the graph nodes,
- size of the multicast group,
- number of the considered criteria.

The first of the above has been chosen in order to reduce the risk of the selected topologies influencing the results too significantly. They are expected to have some impact, so no conclusions should be considered general until confronted with the results for different types of topologies. The following criteria: the number of nodes and the multicast group size are typically used in comparisons [5], [7] and do not require additional explanation. The final variable is one of the improvements of this particular article. As the extension to the previously presented evaluation methodology, the additional dimension of the constraints count is added to the results’ space. Choosing it as one of the variables presents an interesting context of the increasing complexity of satisfying an increasing number of QoS requirements.

Two methods of the topology generation have been selected for the experiment. The Waxman’s [22] and the Barabasi-Albert’s [23] techniques. The numbers: 50, 100 and 150 were selected as the graph sizes. The numbers 2, 3 and 4 were selected for the number of criteria parameters, which reflects a gradual departure from the typical two-criterial comparison. The size of the group was chosen as the main variable and therefore we considered multiple cases of it: 4, 8, 12, 16, 20, 24 and 28.

6. Experiment Results

The experiment results support the claim that the resource drainage evaluation may reveal interesting properties of algorithms. Figures 1–3 present the comparisons of the three algorithms in the Waxman’s graphs of 50, 100 and 150 nodes, respectively. Analogically Figs. 4–6 depict the results for the computations in the Barabasi-Albert’s topologies. In each of the charts, three sets of plots may be seen. One for the Aggregated MLARAC algorithm, one for the Aggregated HMCOP algorithm and one for the HMCMC algorithm. For each of the three, a set of plots is presented for 2, 3 and 4 criteria.

Each of the charts provides evidence that the HMCMC algorithm produces results that are in general the best in most of the cases. However, further details may be observed as well.

First of all, a non-linear characteristics of the HMCMC results in the function of the multicast group size may be observed. Also, the curves present different shapes for a different number of the considered criteria, which shows that the experiment presented in this article revealed previously unknown information. For small multicast groups, the al-
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One of the aggregation based algorithms, the *aggregated HMCOP*, presents comparable performance which may be explained by the fact that it is in principle very similar to the HMCMC at the level of the path finding process. Because the HMCMC approach is optimized in comparison to the aggregation of the HMCOP, and because the final results are similar it may be stated that the HMCMC algorithm turns out better than the aggregated HMCOP with the regard to the assumed comparison criteria.

Different conclusions may be drawn for the *Aggregated MLARAC* algorithm. Firstly, the results tend to be of low quality for greater numbers of the considered criteria, though certain results are still obtained that could potentially present a very good result in the case of the classical success ratio approach. On the other hand, the curve emerging for the low number of the criteria is close to those of the HMCMC.

It is clearly visible that the relationships between different results are very similar in case of both the Waxman’s and Barabasi-Albert’s topologies. They are however different in scale. It can be noticed that some of the phenomena described above are a lot better visible in case of the Waxman’s graphs, especially for the greater amounts of nodes.

A minor conclusion may be therefore made that using different topologies, even if does not change the general comparison result, may contribute significantly to the results readability.

In general, a conclusion may be drawn that for a small number of the criteria and large number of participants, all algorithms present comparable performance, though the HMCMC algorithm is still superior. HMCMC and the aggregated HMCOP results present a non-linear asymptotically decreasing trend, whereas those for the Aggregated MLARAC, though being relatively poor, remain constant. In addition, the HMCMC and the HMCOP algorithms present an interesting instability in relation to the number of considered metrics in the case of small multicast participant groups.

7. Conclusion

The class of the multicriterial constrained multicast routing problems presents a non-trivial level of complexity. Following this concept, a need for a broad analysis techniques spectrum arises. In this article, several of the techniques are described, including a presentation of an innovative technique. The resource drainage comparison presents an interesting extension to the concept of the algorithm success rate analysis, which is supported by the provided interesting and valuable results of the experiments. It has been shown that exploring not only the space of the algorithms, but also the space of their comparison is worth an increased amount of effort as the conclusions may render different algorithms useful in different situations. In addition, the stability of the algorithms against changes in different conditions can be shown with the use of the innovative and non-standard analysis.
References


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