Abstract — This paper is concerned with the issue of optimum detection of known signal in nonwhite noise and/or narrow-band interference. The detection is carried out in three steps. First, some function related to the power spectrum of interfering process \( I(z) \) is estimated via adaptation. Second, the signal + interference is whitened due to \( I(z) \). Third, the replica of signal is filtered via \( I(z) \) to match it to the deformed signal in the previous step. The simulation of SS reception in presence of NB interference shows a high gain in comparison to classical single-matched detection.

Keywords — optimum detection, spread spectrum multiple access systems, NB interference, adaptation.

1. Introduction

The issue of optimum detection is nearly as old as the telecommunication itself [1]. There is a lot of methods and criteria of optimality, e.g. integration or storage, autocorrelation, coherent or synchronous detection, optimum linear or matched filter detection, statistical detection via risk strategies (Bayes, Neyman-Pearson and mini-max) [2 – 4]. Of course, there are some common roots in all methods.

We will confine ourselves to the matched-filter and maximum-likelihood (ML) strategies. In contemporary radio receivers the matched-filter philosophy is widely used but is confined to the useful signal only. It is a reasonable solution for some kind of communications corrupted by AWG noise. In wireless communication, especially spread spectrum (SS) the dominant factor is an outside noise and it diverges considerably from the thermal white model. The resulting miss-adjustment between “white” receiver filter and nonwhite noise/interference causes losses that reach even several tens of dB [5, 6].

This problem has been considered in [7 – 10]. In the first paper [7] an optimum structure of SS receiver is analyzed. Channel distortion and multi-point reception are studied here in detail, whilst colored interference is merely touched on (a simple low-pass case is analyzed). A substantial, but extremely concise approach to nonwhite detection is given in [8]. It contains, however, no functioning structures, no algorithms, and nor comparative results. This gap is partially fulfilled by the author’s studies [9] and [10], which the present contribution is based upon.

The organization of the paper is as follows. In Section 2, the statement of the problem in modern digital form is given. Thereafter a new general detection structure is introduced. In Section 3 and in Appendix A, the adaptive theory, relevant to the matter is provided and the general scheme is complemented and simplified. In Section 4, the simulation of spread spectrum reception in narrow-band interference is carried out and the results obtained are compared with conventional reception.

2. Statement of the problem

As it was shown by many authors [3, 8, 10], the optimum filter for detection of a given signal \( S(\omega) \) in presence of arbitrary noise is

\[
H(\omega) = \frac{S^*(\omega)}{P(\omega)},
\]

where \( S^*(\omega) \) is a conjugate spectrum of useful signal and \( P(\omega) \) — power spectrum of interfering process. The denominator of the desired transfer function, \( P(\omega) \) is usually factorized as \( P(\omega) = H(\omega)H^*(\omega) = |H(\omega)|^2 \) or — within the \( z \) variable – as [8]

\[
P(z) = A^2 G(z) G^*(1/z^*),
\]

where \( AG(z) \) corresponds to \( H(\omega) \), \( AG^*(1/z^*) \) — to \( H^*(\omega) \); \( A \) is a constant; \( z = re^{j\Omega}; -\pi \leq \Omega \leq \pi, r > 0 \) (observe the difference between \( H(\omega) \) and \( H^*(\omega) \)).

\( G(z) \) is said to be a minimum-phase function, while \( G^*(1/z^*) \) — maximum-phase function. As \( G(z) \) is minimum-phase, all its zeros are inside the unit circle and – after inversion – they do not reach the unstable region outside this circle. Hence, the reciprocal of \( AG(z) \) can be easily specified as \( I(z) = 1/AG(z) \) and put in series with the signal matched-filter \( S^*_l(1/z^*) \), see Fig. 1 (\( l \) is an alphabet index).

![Fig. 1. A general idea of double-matched detection](image-url)
The $G(1/z^r)$ is – by definition – maximum-phase and has no reciprocal. However, its inverse Fourier transform $h^*_k$ (said anti-causal response) can be shifted some $N$ steps forward without affecting the transform ($k$ is a discrete time index). This way a causal response $h^*_{k+N}$ is obtained, which – after reversion – can play a role of replica in cross-correlation mode (Fig. 2). Consequently, the co-whitening filter in matched-filter mode stands for a whitening filter in cross-correlation mode, $I_{\text{cowhit}}(z) \rightarrow I_{\text{whit}}(z) = I(z)$ and $h^*_{k+N} \rightarrow h_k$ (Fig. 2).

![Fig. 2. A cross-correlation mode of scheme Fig. 1 for binary signal and unknown interference [8].](image)

An example. Let $P(z) = 1/|1 + 0.5 z^{-1}|^2$. Hence, the whitening and co-whitening functions are: $I(z) = 1 + 0.5 z^{-1}$ and $I^*(1/z^r) = 1 + 0.5 z^{1-r} (A = 1) [8]$. Let an input signal be $s(n) = [1 1 1 \ldots 1]$ for time sequence $n = [-5, 5]$ otherwise $s(n) = 0$. In this case the output signal of $S^*_l(1/z^r)$ will be the same as its input, since we assume $h_l = 1$. The transition of $s(n)$ through $I(z)$ yields $s'(n) = [1 1.5 1.5 \ldots 0.5]$ for $k = [-5, 6]$. The $I^*(1/z^r)$ is maximum-phase, so we factorize it in two functions, $I_1(z) = z^{+1}$ and $I_2(z) = 0.5 + z^{-1}$. Of course, $I(z) = I_1(z)I_2(z)$. The first function $I_1(z)$ shifts $s'(n)$ one step ahead, i.e. $s''(n) = [1 1.5 1.5 \ldots 0.5]$ for $n = [-6, 5]$. The second function transforms $s''(n)$ into $s'''(n) = [0.5 1.75 2.25 \ldots 2.25 2.25 2.25 \ldots 2.25 1.75 0.5]$ for $n = [-6, 6]$. The decision is taken at $n = 0$, hence the result is $S = 2.25$.

Similar operations in cross-correlation mode (Fig. 2), yield $S_z = \sum s''(n)s'(n)\Delta r$, where $s''(n) = s''(n)$ and $\Delta r$ is a time interval between steps. Putting $\Delta r = 1/10$ we obtain $S_z = 2.15$. For a more dense digitalization ($N > 10$) $S_z \approx S$. We will not further deal with the errors appearing in this process, as the signals feeding the modern detectors are digital in nature at the very origin. The same decrements refer to the useful signal as refer to the noise and SNR is held constant.

The scheme Fig. 2 works as follows. An $l$th useful signal $S_{lk}$ and the interference $J_k$ enter the whitening filter $I(z)$. It decorrelates $J_k$ and changes $S_{lk}$. The first action is desirable, as it enables the application of ML principle. The second – is undesirable, so the replica $S^*_{lk}$ is passed through the same filter $I(z)$ to match it to the deformed signal $S_{lk}$. The output products of filters, $y_k$ and $y'_k$ are multiplied one by one and summed up within the range of $k = 1, 2, \ldots, N$. This process is repeated for the full alphabet of signals, $l = 1, 2, 3, \ldots$. The real values of sums are then compared each other and this $l$ is taken as true one, which assigned sum is maximal. At the scheme Fig. 2 we have shown only the simplest binary case for $l = \pm 1$. Now, let us consider the constraints that have been implicitly imposed upon the considered processes.

- First, the power spectrum of interference $P(z)$ has to be determined and/or the whitening stable function $I(z)$ has to exist (even, in approximated form).
- Second, the input signal $S_{lk}$ and the replica $S^*_{lk}$ have to be strictly synchronized one to another and not correlated to the interference $J_k$.
- Third, the interference $J_k$ has to be wide sense stationary (WSS) or at least cycle-stationary (within the adaptive period) and not necessarily Gaussian, whilst possibly nonwhite. The Gaussianity is unnecessary as the interference goes through filter $I(z)$, which normalizes it.

All the above requirements are usually satisfied, except for the first. The power spectrum of interference $P(z)$ and the whitening function $I(z)$ are often unknown, so we will try to get them in a course of adaptation.

### 3. Blind adaptation

The classical adaptive procedure consists in a comparison of a received signal with some standard, e.g. a training sequence. This comparison produces an error, which controls the weights in adaptive filter. In our position, no standard signal exists, so only the blind adaptive algorithms can be considered.

The simple blind algorithm, belonging to the large minimum mean square error group (MMSE) is as follow [10]. At the beginning, an estimate of the interference sample $\hat{y}(n)$ is formed upon its previous states (observations) $y(n-N) \ldots y(n-N+1) \ldots y(n-1)$, Eq. (3). In the next step, this estimate is subtracted from the actual value of $y(n)$ (both are accessible), Eq. (4). The difference $e(n)$ is used for the step by step matching of filter weights $h$ to observed process, Eq. (5). The algorithm is sometimes called least squares (LS) [7, 12]

$$y(n) = \sum_{k=0}^{N} h_k y(n-k) = h^T y,$$

$$e(n) = Ey\left\{[y(n) - \hat{y}(n)]^2\right\},$$

$$\text{h}(n+1) = \text{h}(n) - \mu e(n) y.$$
The step size $\mu$, as well as an initialize sequence $h(0)$ are important data, which affects the convergence and the speed of adaptive process. We experienced to use $h(0) = [1 0 1 0 \ldots]^T$ and $\mu = 0.01 \sim 0.001$. The higher values of $\mu$ correspond to the higher speeds of adaptation and the smaller – to smaller errors of estimation. In any case $\mu_{\text{max}} < 2/\lambda_{\text{max}}$, where $\lambda$ is an eigenvalue of the autocorrelation process.

The estimation time of MMSE algorithms is usually large and the orders of filter – very high. We used the sequences of size $N = 10^3 \sim 3 \cdot 10^3$ and $L = 6 \sim 46$. The higher values of $L$ and $N$ correspond to narrow pulses of interference. For the transmission rates of $10 \sim 30$ kbit/s it stands for an estimation time $\tau = 0.1$ s. More effective algorithms are nonlinear ones, e.g. gradient descent, super exponential and turbo [15]. They use – instead of square error – a notion of cost function. An example of such function is a normalized moment of fourth order:

$$u(4) = \frac{m_4(y)}{m_2(y)} = \frac{(N-L)}{L} \left\{ \frac{1}{N} \sum_{n=L+1}^{N} [y(n) - Y]^4 \right\}, \quad (6)$$

where $y(n)$ is time series of estimated signal; $N$ – series size; $L$ – order of whitening filter.

$$Y = \frac{1}{N-L} \sum_{n=L+1}^{N} y(n), \quad (7)$$

$$m_2(y) = \frac{1}{N-L} \sum_{n=L+1}^{N} [y(n) - Y]^2, \quad (8)$$

$$m_4(y) = \frac{1}{N-L} \sum_{n=L+1}^{N} [y(n) - Y]^4. \quad (9)$$

The moment $u(4)$ for commonly used bipolar signal, $y \in [1, -1]$ takes the lowest value equal 1. In a physical channel, the signal samples interact with one another, which causes cross-correlation (ISI) and gives an increase of $u(4)$. So, if we want to decorrelate (white) the signal, a gradient of $u(4)$ has to be used as an indicator pointing the direction, to which the vector $h$ has to be changed. A weak point of some nonlinear algorithms is their false convergence. If “the spectral channel” of interference is non minimum-phase and reveals several local minima, one of them (not global minimum) can cause a false convergence. In Appendix A, we present the very effective lattice filter and the gradient descent algorithm (GDA) that do not reveal a false convergence.

### 4. Simulation experiments

We are carrying out several experiments to have a deeper insight into the filtration and detection processes and to estimate a gain of the method. We used the spread spectrum signal of the form

$$S_{l,k} = l \times [s_1, s_2, \ldots, s_N], \ldots, l \times \left\{ \sum_{s_{k+1} \neq k \mod N} s_{k+1} \right\}_{l \mod N}, \ldots, l \times \left\{ s_{MN-N+1}, \ldots, s_{MN} \right\}_{l \mod N}, \quad (10)$$

where $l$ is a binary random number, $l \in [1, -1]$, its sequence $\{l\}$ contains the information carried on; $N$ is a spreading factor; $s_{k} = s_{k+1}, \ldots, s_{k+q}$, $k \mod N \neq 1$ represents $m$th bit of signal, $m = 1, 2, \ldots, MN$; $s_k$ is kth sample of signal, $k = 1, 2, \ldots, MN$; $s_k \in [1, -1]$; $\gamma$ – real constant; the sequence $s_k$ is chosen according to Gold code and is known to the sender and recipient of information.

The appropriate interference sequence is $J_k = \{J_1, \ldots, J_{MN}\}$ where $MN$ is a sequence size in chips. The replica $S_{l,k}$ follows the useful signal except for the attributes of power ($\gamma$), information carried on ($l$), and conjugation index (*). $s_{l,k}^* = |y_{l,k}^*|$.

We have carried out four experiments. The first one is concerned with the white noise case. The symbol of sum in Fig. 2 denotes

$$\sum_{N = 1}^{N} \sum_{k = 1}^{jN} y_k y_{l,k}, \quad j = 0, 1, 2, \ldots, M - 1, \quad (11)$$

where $y_k$ and $y_{l,k}$ are the outputs of whitening filters in the signal and replica leads, respectively.

The obtained BER curve is shown in Fig. 3 as the base curve (0). We found that interference other than white, e.g. low-pass of $H(z) = 1/(1 - 1.4 z^{-1} + 0.5 z^{-2})$ causes a degradation of reception, expressed by curve (1). This is contrary to Shannon’s theory, which states that white noise causes the highest degradation of reception. So, we insert a proper whitening filter into both paths of detector, $I(z) = 1/H(z) = 1 - 1.4 z^{-1} + 0.5 z^{-2}$ (experiment II). The symbol of sum denotes now

$$\sum_{N = 1}^{N} \sum_{k = 1}^{jN} y_k y_{l,k}, \quad (12)$$

where $L$ – order of filter ($= 2$).

The obtained BER curve is shown as curve (2), see Fig. 3. We observe that it has been shifted about 7 dB down the base curve (0). This is a striking difference and it means that the single-matched (white) detector does not recognize the color of interference.

Then, further experiments were carried out. In the third one, III, an interference pulse of natural signal was used. Let its autocorrelation function and power spectrum be (e.g. BPSK signal)

$$R(\tau) = 1 - |\tau|/T \text{ for } |\tau|/T < 1, \text{otherwise } R(\tau) = 0, \quad (13a)$$

$$P(\omega) = |T \sin(\omega T/2)/(\omega T/2)|^2. \quad (13b)$$

Having $P(\omega)$ we can determine its corresponding transfer function $H(\omega)$ and the unit impulse response $h$ via Yule-Walker method [18]

$$[a, b] = \text{yulewalk}(L, F, M), \quad h = \text{impz}(a, b), \quad (14)$$
adaptation we obtained activated (dotted line block, Fig. 2). In a course of MMSE ence spectrum is not known, so, the adaptive loop was assumed that interfer-
ing double pulses) were examined (Table 1). All results were very good [8]. Several experiments have been carried out using the lattice filters and the gradient descent algo-

The formulae for $H(z)$ and its coefficients are

$$H(z) = \frac{a_0 + a_1z^{-1} + \ldots + a_{16}z^{-16}}{1 + b_1z^{-1} + \ldots + b_{16}z^{-16}},$$

where $L$ is a chosen order of filter; $F$ – frequency scale vector, e.g. $F = [0 0.01 \ldots 1]^T$; $M$ – interference magnitude vector, $M = |H(F)|^T$; $a, b$ – transfer function coefficients vectors.

The obtained results are comparable to the findings of other authors [5, 6]. The method is intended for systems with interference dominating inner thermal noise and be-

In spite of so rigorous theoretical limit, the practical re-

Figure 3. A family of BER curves for single-matched and double-

matched detection at different interference.

Fig. 4. A structure of whitening lattice filter (see Appendix A).

It should be also noted that a described process is strictly optimal only for known interference spectrum. Otherwise, when using adaptation, the algorithm “learns” the spectrum and its “equivalent channel” is non-minimum-phase and the whitening filter is FIR, then the detector can only be referred to as asymptotically optimum (for $L \to \infty$) [12]. In spite of so rigorous theoretical limit, the practical re-

![Figure 3](image3.png)

![Figure 4](image4.png)

5. Conclusion

This paper presents a new method of spread spectrum de-

tection in presence of nonwhite noise and/or narrow-band inter-

ference. It uses a double-matched detector, fitted both to the useful signal and interference. It is formed by adding a pair of whitening filters and a blind adaptive loop to the conventional single-matched ML structure. Simulation of base-band SS reception under typical NB interference yields the gain approaching 20 dB for the signal-to-

Interference bandwidth ratio $B/B_i \approx 10$. Unfortunately, the gain reduces to zero for flat interference.

The obtained results are comparable to the findings of other authors [5, 6]. The method is intended for systems with interference dominating inner thermal noise and being ruggedly nonwhite. It can be treat as a compliment to RAKE and other upgrading techniques.
Appendix A

Gradient descent algorithm and the lattice filter

The normalized moment \( u(4) \) and its derivatives are

\[
u(4) = \frac{m_4(y)}{m_2^2(y)}, \quad \frac{du}{dy} = \frac{dm_3}{dy} \frac{dm_2}{dy} K = [K_1, K_2, \ldots, K_L]^T, \tag{1A}\]

were \( K \) is so-called reflection coefficient in lattice filter, see Fig. 4 (it corresponds to the weight vector \( \mathbf{h} \) in transversal FIR filter); \( y \) is an output signal of filter. Taking into account Eqs. (12) and (13) in main text, we obtain (2A)

\[
\frac{du}{dy} = \frac{4m_4}{m_2^2} \sum \left[ y(n) - y \right]^3 + \frac{4m_3}{m_2^2} \sum \left[ y(n) - y \right] = U - V. \tag{2A}\]

As it comes from Fig. 4 (derivation in [11])

\[
\frac{dy}{dk_1} = s(n-1)(1 + K_1) + s(n-2)(0 + K_1) + s(n-3)(0 + \ldots, \tag{3A}\]

\[
\frac{dy}{dk_2} = s(n-1)(K_1 + K_3) + s(n-2)(1 + K_4) + s(n-3)(0 + \ldots, \tag{4A}\]

\[
\frac{du}{dK_p} = \left\{ U - V \right\} \left\{ s(n-1) [K_{p-1} + K_{p+1}] + s(n-2) [K_{p-2} + K_{p+1}] + \ldots \right\} = \left\{ U - V \right\} \sum_{i=1}^{L} s(n-i) [K_{p-i} + K_{p+1}], \tag{5A}\]

where \( s(\cdot) \) is input signal of the filter, and

\[
K_0 = 0, \quad K_{p+i} = 0 \quad for \ p + i < 0 \ or \ p + i > L. \tag{5A}\]

The quantities \( U, V \) are given in (2A). The fundamental gradient equation is

\[
\mathbf{K}(n+1) = \mathbf{K}(n) - \mu \nabla_{\mathbf{K}} u(4), \tag{6A}\]

\[
\nabla_{\mathbf{K}} u(4) = \begin{bmatrix} \frac{du(4)}{dK_1} & \frac{du(4)}{dK_2} & \ldots \end{bmatrix}^T, \tag{7A}\]

where \( \mu \) is a step size and \( n \) – iteration number. The convergence criteria, initialize parameters and a step size \( \mu \) are considered in [12, 15].

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