Localization in Wireless Sensor Networks Using Heuristic Optimization Techniques

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Abstract—Many applications of wireless sensor networks (WSN) require information about the geographic location of each sensor node. Devices that form WSN are expected to be remotely deployed in large numbers in a sensing field, and to self-organize to perform sensing and acting task. The goal of localization is to assign geographic coordinates to each device with unknown position in the deployment area. Recently, the popular strategy is to apply optimization algorithms to solve the localization problem. In this paper, we address issues associated with the application of heuristic techniques to accurate localization of nodes in a WSN system. We survey and discuss the location systems based on simulated annealing, genetic algorithms and evolutionary strategies. Finally, we describe and evaluate our methods that combine trilateration and heuristic optimization.

Keywords—evolutionary strategy, genetic algorithm, localization, location systems, nonconvex optimization, simulated annealing, wireless sensor network.

1. Introduction

The primary function of a location estimation method is to calculate the geographic coordinates of network nodes with unknown position in the deployment area. Most applications of wireless sensor networks (WSN) require the correlation of sensor measurements with physical locations, even if the accessible knowledge about positions of nodes is only approximate. Moreover, information about current locations are used in geographical-based routing, data aggregation and various network services. Hence, self-organization and localization capabilities are one of the most important requirements in sensor networks.

Information on the location of nodes can be obtained in two ways:

- recording data on the location of nodes during their distribution,
- fitting nodes with a GPS system.

Both methods have significant defects. Typical WSN usually consists of a large number of sensors that should be densely distributed in a sensing field. The large number of nodes usually precludes manual configuration. Moreover, manually recording and entering positions of each sensor node is impractical and impossible in many applications, in which sensors are distributed randomly in ad hoc fashion, which is cheaper, and in some cases the only possible solution. Moreover, this method cannot be used in mobile networks where nodes can travel. Another solution is to collect data on the location of sensors by means of GPS devices. This solution can be used in different types of networks, including mobile ones. Unfortunately, it is very costly, both due to the price of GPS receivers, and to the increased requirements related to power consumption that may decrease the lifetime of a WSN. Moreover, adding an additional receiver increases the size and weight of the total device (network node).

Due to the drawbacks of presented solutions, many automated location systems for assigning geographic coordinates to each node have been developed. All these schemes should work with inexpensive off-the-shelf hardware, minimal energy requirements, scale to large networks, and also achieve good accuracy in the presence of irregularities and give the solution in the short time. Various localization strategies for WSNs have been developed [1]–[4]. Position calculation can be conducted using one machine collecting data from the network (base station) or calculations can be distributed. In centralized schemes, data collected in a network is transmitted to the central machine that calculates the positions of nodes with unknown location. Distributed algorithms relay only on local measurements – each non-anchor node estimates its position based on data gathered from its neighbors.

The main contribution of this paper is to provide a survey of localization techniques and systems using nonconvex, heuristic optimization techniques to solve the localization problem. We focus on centralized schemes with heuristic optimization, and present the location systems based on simulated annealing, genetic algorithms and evolutionary strategies.

The paper is organized as follows. The introduction to localization techniques is provided in Section 2. The localization problem is formulated in Section 3. Strategies and location systems based on heuristic optimization are investigated in Sections 4 and 5. In Section 6 we evaluate two our location systems TGA and TSA. The paper concludes in Section 7.

2. Localization Techniques

A number of localization methods and location systems are described in the literature. A general survey is found
in [1]–[4]. The localization techniques can be classified with respect to various criterion. They differ on network architecture and configuration, hardware components, nodes properties and deployment, measurement and calculation methods, computing organization, assumed localization precision, etc. Recently proposed localization techniques consist in identification of approximate location of nodes based on merely partial information on the location of the set of nodes in a sensor network. Let us consider a network formed by \( L = M + N \) sensors; \( M \) anchor nodes and \( N \) non-anchor nodes. The definitions of anchor and non-anchor nodes are as follows:

**anchor node** – a node that is aware of its own location, either through GPS or manual recording and entering position during deployment. Its position is expressed as \( n \)-dimensional coordinates \( a_k \in \mathbb{R}^n \), \( k = 1, \ldots, M \).

**non-anchor node** – a node that is not aware of its own location in the deployment area. Its position is expressed as \( n \)-dimensional coordinates \( x_j \in \mathbb{R}^n \), \( j = 1, \ldots, N \).

The goal of a location system is to estimate coordinate vectors of all \( N \) non-anchor nodes. In general, localization schemes operate in two stages:

**Stage 1**: Inter-node distances estimation based on hop connection information (hop counting) or true physical distance calculation based on inter-node transmissions and measurements.

**Stage 2**: Transformation of calculated distances into geographic coordinates of nodes forming the network.

### 2.1. Stage 1: Inter-node Distance Estimation

With regard to hardware’s capabilities of given nodes, and the mechanisms used for estimating inter-node distances in **Stage 1** of the localization scheme, we divide the localization algorithms into two categories: range-free (connectivity-based) methods and range-based (distance-based) methods.

The **range-free** algorithm uses only connectivity information to locate the entire sensor network. The popular solutions are hop-counting techniques. Assume that each anchor node \( a_k \), \( k = 1, \ldots, M \) exchanges messages with other nodes. Hence, the distances in hops \( h_{kl} \) between each pair \((k,l)\) of anchors in the network are estimated. Next, each anchor computes an average size for one hop \( c_k = \frac{\Sigma_{s \in S_k} ||a_s - a_0||}{\Sigma_{s \in S_k} h_{su}} \), \( k \neq l \), where \( S_k \) denotes a set of anchors located within a transmission range of \( r_k \), \( S_k = \{(k,l) : ||a_k - a_l|| \leq r_k\}, l = 1, \ldots, M \). The calculated values are broadcasted into the network, and the inter-node distances expressed in hops are estimated.

The **range-based** algorithm uses absolute point-to-point distance estimates (range) or angle estimates in location calculation. Hence, distance-based methods require the additional equipment but through that we can reach much better resolution than in case of range-free ones. In accordance with the available hardware they exploit Angle of Arrival (AoA), Time of Arrival (ToA), Time Difference of Arrival (TDoA) and Received Signal Strength Indicator (RSSI). The survey and discussion of the most popular measurement technologies is available in [1], [5]–[9]. The common technique based on a standard feature found in most wireless devices is RSSI. The method for distance estimation based on a signal propagation model and RSSI is described in [10]. The advantage of this method is low cost (no additional hardware), easy configuration, calibration and deployment. The disadvantage is low level of measurement accuracy because of high variability of RSSI value. In real-world channels, multipath signals and shadowing are two major sources of environment dependence in the measured RSSI.

### 2.2. Stage 2: Geographic Coordinates Estimation

In **Stage 2** of the localization scheme the calculated distances are converted into geographic coordinates of network nodes. Different less and more complicated techniques may be used to perform calculations. The coordinates of nodes can be calculated using: geometrical techniques, multidimensional scaling, stochastic proximity embedding, optimization algorithms (nonlinear, quadratic and linear), hybrid schemes that use two different techniques.

The **geometrical techniques** give solutions to a set of nonlinear equations. The most popular are: triangulation, trilateration and multitrilateration. The simple and popular localization system implementing the trilateration method is called Ad-hoc Positioning System (APS). It is described in [11]. **Multitrilateration** methods are proposed to reduce limitations of the typical trilateration scheme. **Atomic multitrilateration** incorporates distance measurements from multiple neighbors. The idea of **iterative multitrilateration** is to repeat trilateration for increased number of anchor nodes (every iteration each non-anchor node with estimated position changes its role to anchor). The philosophy of localization techniques based on multidimensional scaling (MDS) and stochastic proximity embedding SPE is to transform a mathematical model to convert distance information into the coordinate vector. The common idea of other methods is formulating the localization problem as a nonlinear, nonconvex optimization task solved by global optimization (often heuristic) solvers or relaxing the resulting problem as a convex optimization problem solved by quadratic or linear solvers. Recently, a popular group consists of hybrid systems that use more than one technique to estimate location, i.e., results of initial localization are refined using another localization method.

The survey, evaluation and detailed discussion of the most popular approaches to geographical coordinates estimation and location systems are found in [1], [3], [7], [11]–[14]. In this paper we present a short overview of location systems using heuristic techniques. We start our presentation from the formulation of the mathematical model of the WSN localization problem in Section 3.
2.3. Flip Ambiguity Phenomenon

In many WSN applications it can be observed that some nodes can not be uniquely localizable. These location errors are often driven by so-called flip ambiguity phenomenon, demonstrated in Fig. 1. As the neighbors of node $D$ are almost collinear, and the inter-node distances are estimated with measurement errors the localization algorithm usually calculates the incorrect location, i.e., $D'$ instead of $D$ in Fig. 1. It is obvious, the position of this node can be reflected with no significant change in the performance function in Eq. (1). This observation is discussed by many researchers, and different methods to solve this problem are proposed.

![Fig. 1. Flip ambiguity phenomenon in WSN localization.](image)

The popular approach to compensate location errors driven by a flip ambiguity phenomenon is to modify the basic localization algorithm or extend the localization process in a correction phase.

3. Mathematical Model of WSN Localization Problem

The standard approach is to formulate a localization problem as the optimization task with the nonlinear performance function $J_N$:

$$
\min_{\hat{x}} J_N = \sum_{k=1}^{M} \sum_{j \in S_k} (\hat{d}_{kj} - d_{kj})^2 + \sum_{i=1}^{N} \sum_{j \in S_i} (\hat{d}_{ij} - d_{ij})^2,
$$

(1)

where $\hat{d}_{kj} = ||a_k - \hat{x}_j||$, $\hat{d}_{ij} = ||\hat{x}_i - \hat{x}_j||$, $a_k$ denotes the real position of the anchor-node $k$, $\hat{x}_i$ and $\hat{x}_j$ denote, respectively, the estimated positions of nodes $i$ and $j$. $d_{kj}$ and $d_{ij}$ the estimated distances between pairs of nodes calculated based on measurements, and $S_i$, $S_k$ sets of neighboring nodes defined as follows:

$$
S_k = \{ (k, j) : ||a_k - x_j|| \leq r_k, \quad j = 1, \ldots, N \}
$$

$$
S_i = \{ (i, j) : ||x_i - x_j|| \leq r_i, \quad j = 1, \ldots, N \},
$$

where $x_i$ and $x_j$ denote real positions of nodes with unknown locations and $r_i$ and $r_k$ their transmission ranges.

Various optimization techniques are used to solve the optimization problem Eq. (1). As it was mentioned the most popular approaches are: quadratic programming, linear programming, nonlinear and nonconvex optimization techniques. The first class of methods transforms the original nonconvex formulation Eq. (1) into quadratic problem and apply quadratic programming to solve the reformed problem. A localization system OPDMQP using quadratic programming is described in [15]. The second class of methods relaxes the problem (1) in order to obtain a semidefinite programming SDP [12] or a second-order cone programming SOCP [16]. The existing linear solvers (usually interior point methods) are used to solve the transformed problem. In case of both mentioned approaches the problem with local minima can influence the solution. The third, commonly used strategy is to apply global optimization algorithms to avoid local minima of the performance function $J_N$ in Eq. (1). Numerous approaches are proposed and described in the literature. Many researchers suggest to use popular heuristic methods, i.e., deterministic, such as tabu search TS and stochastic, such as simulated annealing SA, genetic algorithm GA, evolutionary algorithm EA or particle swarm optimization PSO to calculate the location estimates.

4. Location Systems with Heuristic Optimization – A Survey

In this section we discuss selected location systems using heuristic optimization.

4.1. Simulated Annealing based Systems

Results of simulated annealing to location estimation are provided in several papers. The simulated annealing based localization system (SAL) developed by Kannan et al. is described in [13]. It is the range-based system. The authors propose different modifications of basic SA to improve the results and speed up calculations. They show that for ideal measurements without any noise introduced to the system when inter-node distances are calculated with 100% accuracy the location estimates are computed with 100% accuracy, too. The measurement noise influences the results but they are quite accurate. The serious deterioration of results is observed in case when flip ambiguity situation occurs. To compensate location errors driven by the flip ambiguity phenomenon Kannan et al. propose a method [14], in which the localization is done through two phases, i.e., in the first phase the coordinate vectors are calculated (localization phase), in the second phase the errors caused by the flip ambiguity are compensated (refinement phase). Hence, two executions of the simulated annealing method are performed. The goal of the first execution is to solve the optimization problem Eq. (1), and calculate the coordinates of the target nodes. The second phase is performed only on non-uniquely localizable nodes. The goal of this phase is to identify these nodes, and refine their location.
estimates calculated in the first phase. The SA algorithm is used again to solve the optimization problem with modified objective function defined in Eq. (3). The function value is increased when a node is placed in a wrong neighborhood.

\[
J_{F_k} = \sum_{k=1}^{K} \left( \sum_{j \in S_k} (\hat{d}_{kj} - d_{kj})^2 + \sum_{j \in S_k \delta_{kj} \leq r_k} (\hat{d}_{kj} - r_k)^2 \right) + \sum_{i=1}^{N} \left( \sum_{j \in \tilde{S}_i} (\hat{d}_{ij} - d_{ij})^2 + \sum_{j \in \tilde{S}_i \delta_{ij} \leq r_i} (\hat{d}_{ij} - r_i)^2 \right),
\]

(3)

where \( r_k \) and \( r_i \) denote the transmission ranges of the nodes \( k \) and \( i \).

In summary, the goal of the localization phase is to calculate the accurate coordinate vectors of uniquely localizable nodes and initial coordinate estimates of non-uniquely localizable nodes. The goal of the refinement phase is to increase the accuracy of the location estimation of all non-uniquely localizable nodes.

4.2. Genetic Algorithm and Evolutionary Strategy based Systems

Another approach is to use various versions of genetic algorithm or evolutionary algorithm. The genetic algorithm based localization system (GAL) developed by Zhang et al. is described in [17]. The authors propose different modifications of basic GA to improve the results and speed up calculations. Two new genetic operators are adopted: a single-vertex-neighborhood mutation and a descend-based arithmetic crossover. The method was evaluated on several example problems. The authors claim that it outperforms the SDPL method (a semi-definite programming with gradient search localization) and simulated annealing based localization SAL [13].

The range-based localization system with the distances estimation based on RSSI and Imperialist Competitive Algorithm (ICA) used to calculate the coordinate vectors is presented by Sayadnavard et al. ICA is a new evolutionary algorithm that is based on the simulation of a human’s socio-political evolution. The simulation results presented in [18] highlight that ICA-based approach considerably outperforms the APS system. Moreover, it calculates estimates characterized by higher accuracy than the ones obtained by the PSO-based localization scheme using RSSI ranging technique [19] but with more computational time.

The application of evolutionary computation to estimate locations of nodes is described in [20]. The estimates of inter-node distances calculated due to RSSI measurement are transmitted to the central unit. The central unit employs evolutionary algorithm to estimate the locations of nodes based on gathered information about inter-node distances. The authors claim that their approach gives a reasonable solution even for poor RSSI measurement but they do not provide any comparison to solutions developed by other researchers. Moreover, they do not consider the flip ambiguity situation.

Vecchio et al. developed a location system, in which two-objective localization problem is formulated and evolutionary algorithm is used to solve it [21]. Due to the fact that the connectivity in WSN is not sufficiently high the authors propose some modifications to a basic EA. The algorithm takes into account both the localization accuracy and certain topological constraints induced by connectivity considerations during a location estimation. In proposed scheme two performance functions are concurrently minimized. The first one is defined in Eq. (1). The second cost function is defined as follows:

\[
J_{F_k} = \sum_{k=1}^{K} \left( \sum_{j \in S_k} \delta_{kj} + \sum_{j \in S_k \delta_{kj} \leq r_k} (1 - \delta_{kj}) \right) + \sum_{i=1}^{N} \left( \sum_{j \in \tilde{S}_i} \delta_{ij} + \sum_{j \in \tilde{S}_i \delta_{ij} \leq r_i} (1 - \delta_{ij}) \right),
\]

(4)

where \( \delta_{ij} = 1 \) if \( \hat{d}_{ij} > r_i \) and 0 otherwise, and \( \tilde{S}_i^c = \{ (i, j) : ||x_i - x_j|| > r_i \} \). Hence, the goal of the \( J_{F_k} \) function is to count the number of connectivity constraints that are not satisfied by the current estimated locations of target nodes. The authors claim that their approach outperforms the SA-based localization algorithm proposed in [14]. The simulation results presented in [21] confirm the good performance of the algorithm.

5. Hybrid Methods

In this section we investigate selected systems using at least two different methods to calculate estimates of nodes location in a network. Moreover we describe our methods that combine geometrical techniques along with a heuristic optimization.

5.1. Hybrid Methods – A Survey

The last presented strategies are hybrid schemes that combine commonly used methods for computing geographic coordinates of nodes. In most approaches trilateration or multilatiration is used to calculate an initial solution, which is improved in the next step. Tam et al. developed a two-phase method that is described in [22]. The APS system based on the basic trilateration is used to calculate the initial localization. The micro-genetic algorithm (MGA) is adopted to improve the accuracy of calculated estimates. The application of APS and MDS-based algorithm is proposed in [23].

Shekofteh et al. propose the localization scheme TS&SA, in which two different optimization methods executed in cascade are used to estimate locations of network nodes. The scheme is described and evaluated in [24]. It operates in two phases. Tabu search (TS) is executed in the first phase to solve the optimization problem Eq. (1) and estimate initial locations of node. In the second phase the simulated annealing (SA) method is used to refine the location estimates of all non-uniquely localized nodes. Similarly to Kannan et al. method described in [14] the optimization problem with the cost function \( J_{F_k} \) Eq. (3)
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Fig. 2. Phase 1: calculating the initial solution using multitrilateration.

is solved to compensate localization errors. The method was evaluated through simulations. The authors claim that the TS&SA-based location system has better convergence characteristics compared to the SA-based system described in [13], but in the cited paper only the results of the TS&SA system simulation are demonstrated and discussed without comparison to other solutions, especially systems in which location errors driven by the flip ambiguity phenomenon are compensated.

We developed a hybrid scheme to location calculation that combines iterative multilateration along with nonconvex optimization and final correction. Two versions of this scheme are available: TSA: Trilateration & Simulated Annealing and TGA: Trilateration & Genetic Algorithm, and described in [25]. Both algorithms are range-based with RSSI technique used to distances estimation.

5.2. TSA and TGA Methods

TSA and TGA methods operate in two phases. In the beginning of the first phase all nodes in the network are divided into two sets: \( A = \{a_1, \ldots, a_M\} \) containing anchor
nodes, and \( B = \{ x_1, \ldots, x_N \} \) of nodes with unknown location. Next, iterative multilateration is used to determine the relative positions of nodes from the set \( B \) based on the known locations of nodes from \( A \), and the estimated distances between pairs of nodes. To determine the relative positions of each non-anchor on a 2D plane using trilateration at least three neighbors with known locations are needed. In every iteration each node from \( B \) with estimated position is moved to the auxiliary set \( C \) and finally, in next iteration of the algorithm changes its role to anchor and move to \( A \). This phase stops when there are no more nodes from \( B \) that can be localized based on the available information about their neighbors. Figure 2 shows the performance of the phase 1.

In the second phase the optimization problem Eq. (1) is formulated, and the SA or GA algorithm is used to solve it. The goal of this phase is to increase the accuracy of the location estimation calculated in the first phase, and estimate the position of nodes that cannot be calculated using iterative multilateration. The implementations of simulated annealing SA and genetic algorithm GA applied to TSA and TGA schemes are described below.

Simulated annealing method was implemented in TSA according to the algorithm described in [13]. It is a classical version of SA with one modification – the cooling process is slowed down. At each value of the coordinating parameter \( T \) (temperature), not one but \( q \cdot N \) non-anchor nodes are randomly selected for modification (where \( N \) denotes the number of non-anchors in the network and \( q \) is a reasonably large number to make the system into thermal equilibrium). Coordinate estimations of chosen nodes are perturbed with a small displacement of the distance \( \Delta d \) in a random direction. The structure of the SA algorithm is presented in Fig. 3.

The algorithm consists of the following elements and operations:

**Task configuration.** The goal of this task is to localize \( N \) non-anchor nodes in a network. The initial location of all nodes is determined in phase 1 of the algorithm.

**Moving operation.** In each iteration of the algorithm a new solution is calculated. The node is randomly selected and is moved in random direction at distance \( \Delta d \). The value of \( \Delta d \) depends on the control parameter \( T \) – the distance \( \Delta d \) is restricted by shrinking factor \( \beta < 1 \), \( (\Delta d)^{new} = \beta \cdot (\Delta d)^{old} \).

**Performance measure.** The performance measure is defined in Eq. (1).

**Cooling scheme.** The simple cooling scheme is proposed: \( T_{new} = \alpha \cdot T_{old} \).

A classical version of a genetic algorithm GA was applied to TGA. It applies the following operators:

**Task configuration.** The goal of this task is to localize \( N \) non-anchor nodes in a network. The abstract representation of candidate solutions called chromosomes are vectors of random variable – coordinates of all non-anchor nodes: \( [x_1, x_2, \ldots, x_N] \), \( x_i \in \mathbb{R}^n \).

**Initial population.** The initial population consists of \( N \) chromosomes, the genes of which (initial coordinates of all nodes) were determined in the first phase of the algorithm.

**Performance measure.** Similarly to SA algorithm the performance measure (fitness function) is defined in (1).

**Selection.** The tournament selection of size \( q = 2 \) is used.

**Crossover.** Discrete recombination similar to elements exchanging applied to binary vectors is used with one modification – all coordinates of a given node are recombined simultaneously.

**Mutation.** The simple mutation operator is used. The components of chromosome are modified by adding a vector of generated \( 2N \) Gaussian random variables.

The method for correction of incorrect location estimates driven by the flip ambiguity phenomenon is provided in the second phase of TSA and TGA. Our submission is to use nested optimization to solve a problem with non-uniquely localizable nodes. The idea is to introduce the additional functionality – the correction operation to the optimization solver. The correction is triggered every iteration in the optimization process whenever the value of the performance function \( J_N \) defined in Eq. (1) is lower than a threshold \( \theta \). Trilateration is executed to relocate all nodes placed
in wrong neighborhoods by exploiting the nodes violating a smaller number of neighborhood constraints than the other randomly selected nodes. The threshold $\theta$ depends on the number of anchor nodes, network density and deployment, power of radio devices and expected noise measurement factor $n_f$. It is tuned according to the following formula:

$$\theta = \begin{cases} \mu \cdot n_f \cdot s^2, & \frac{N+M}{M} < \gamma \\ \lambda \cdot n_f \cdot s^2, & \frac{N+M}{M} \geq \gamma \end{cases}$$  \hspace{1cm} (5)

where $n_f$ is the noise measurement factor, $\mu$, $\lambda$ and $\gamma$ experimentally tuned parameters. The variable $s$ denotes an average number of neighbors of all nodes forming a network:

$$s = \frac{1}{N+M} \sum_{i=1}^{N+M} \sum_{j \in S_i} c_{ij},$$  \hspace{1cm} (6)

where $c_{ij} = \begin{cases} 1, & j \in S_i \\ 0, & j \notin S_i \end{cases}$

where $c_{ij}$ denotes the connectivity between $i$ and $j$ nodes, and $S_i$ a set of neighbors of the node $i$. The correction algorithm is described in details in [25].

6. Tests and Evaluation

We validated selected location systems through simulation. All tests were performed on Intel Core2 Duo E6600 – 2.4 GHz, 2 GB RAM using our simulator, which employs Link Layer Model for MATLAB described in [26] for network model generation. The goal of all tests was to compare the accuracy and robustness of various approaches to the coordinate vector calculation. To evaluate the accuracy of tested location systems we used the mean error between the estimated and the true physical location of the non-anchor nodes in the network defined as follows:

$$LE = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{| |\hat{x}_i - x_i| |^2}{r_i^2} \right) \cdot 100\%,$$  \hspace{1cm} (7)

where $N$ denotes the number of nodes in a network, which location is estimated, $LE$ denotes a localization error, $x_i$ the true position of the node $i$ in the network, $\hat{x}_i$ estimated location of the node $i$ (solution of the location system) and $r_i$ the radio transmission range of the node $i$. The localization error $LE$ is expressed as a percentage error. It is normalized with respect to the radio range to allow comparison of results obtained for different size and range networks.

All evaluated methods were range-based with inter-node distances calculated due to RSSI. We performed simulations for a network formed by 200 nodes (20 anchors and 180 non-anchors). Nodes distribution in a deployment area is presented in Fig. 4. Different network topologies were considered in our experiments. We analyzed the impact of network density and RSSI measurement errors on the accuracy of the location estimation. The density was expressed by a connectivity measure that was defined as an average number of neighbors of all nodes in a network. Two levels of inter-node distance estimation error involved by RSSI measurement errors were considered. To convert the measurements into the inter-node distances we applied the radio channel model. The algorithm is described in [10]. The detailed information about radio channel modeling can be found in [26]. Finally, we performed six series of experiments for various network density and measurement errors:

- connectivity measures: low (7–8 neighbors), medium (13–14 neighbors), high (20–21 neighbors),
- levels of distance estimation error: low (LDEE) with 0% – 0.2% error, high (HDEE) with 15% – 20% error.

The assumed distance estimation errors (in percent) for low, medium and high density networks are collected in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Distance estimation error</th>
<th>Connectivity [%]</th>
</tr>
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<tbody>
<tr>
<td>LDEE</td>
<td>0.07</td>
</tr>
<tr>
<td>HDEE</td>
<td>15.83</td>
</tr>
</tbody>
</table>

It is obvious that lower network density usually involves increasing number of weekly connected nodes. The presence of unconnected or weekly connected nodes in a network has significant impact on localization error Eq. (7). We tested the influence of the weekly connected node on

![Fig. 4. Nodes deployment.](image)
the location estimation (a node marked with filled circle in Fig. 4); tests for low density network).

The localization problems formulated for networks defined in Table 1 were solved using three location systems: SAL, TSA and TGA. Tables 2 and 3 present the localization errors obtained respectively, for low and high measurement errors, and low, medium and high connectivity. From the experimental results we can observe that the best localization accuracy was obtained using the TSA algorithm both for low and high measurement errors. The difference in localization quality is especially visible for low density networks. For higher density network the solutions calculated using the SAL system are satisfactory. In case of TGA the best results can be obtained for high density networks with low measurement error, and these results are almost as good as for TSA. It should be noted here that for low and medium connectivity it was impossible to calculate the accurate location (with 0% error) because of presence of the weekly connected node (see Fig. 4).

Simulation results confirm that TSA is an efficient and robust localization method. Using the TSA method we calculated the most accurate location estimates for all tested networks with the smallest standard deviation of the solutions. However, it should be underlined that efficiency and robustness of localization methods using heuristic techniques strongly depend on different control parameters of the algorithm. To design the general purpose algorithm to solve the localization problem the parameters should be tuned for various network size and topology. The TSA method was exhaustively tested on different networks in order to tune control parameters. It is very probable that both SAL and TGA methods can be tuned up to guarantee better accuracy, however this process is time consuming with no guarantee of success.

### 7. Summary and Conclusions

Sensor network localization continues to be an important research challenge. In this paper a short survey of the localization strategies and systems using global optimization methods is presented. We focus on application of heuristic techniques, such as simulated annealing, genetic and evolutionary computation. Referring to the literature and considering results of our research it seems that optimization methods, such as SA, GA, EA considerably outperform systems based on linear or quadratic programming (SDP, SOCP, QP). In most tests described in literature heuristic algorithms gave an acceptable location accuracy in an acceptable computation time. Our experimental results presented in this paper demonstrate that the hybrid techniques are competing to the other solutions. Systems that combine geometrical and non-convex optimization techniques extended with correction of temporary solutions provide significant robustness and improve an accuracy compared to a simple trilateration, convex and simple non-convex optimization. Hence, from the perspective of location estimation accuracy the suggestion is to use centralized range-based hybrid location systems with measurement techniques according to the available hardware and additional correction of localization errors. In summary, we can say that sensor network localization continues to be an important research challenge. Despite, many methods and systems to estimate the location of nodes in WSN are proposed and described in literature, development of robust, accurate and scalable location system is still a challenging task.

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